

DIFFERENTIAL CALCULUS
 FINAL Review
 2+2 Math 2009

NAME: _____

1. Let

$$g(x) = \begin{cases} -x & \text{if } x \leq -1 \\ 1-x^2 & \text{if } -1 < x < 1 \\ x-1 & \text{if } x > 1 \end{cases}$$

Evaluate each of the following limits, if it exist.

(i) $\lim_{x \rightarrow 1^+} g(x)$

(ii) $\lim_{x \rightarrow 1} g(x)$

(iii) $\lim_{x \rightarrow 0} g(x)$

(iv) $\lim_{x \rightarrow -1^-} g(x)$

(v) $\lim_{x \rightarrow -1^+} g(x)$

(vi) $\lim_{x \rightarrow -1} g(x)$

2. Let $f(x) = \left(\frac{1+3x}{1+4x^2+3x^4} \right)^3$ find $\lim_{x \rightarrow 1} f(x)$

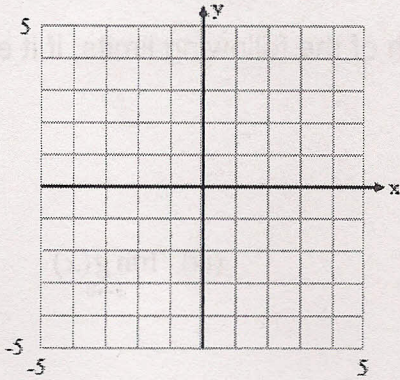
3. A) Find the limit $\lim_{x \rightarrow 1} \frac{2-x}{(x-1)^2}$

B) Is f continuous at $x = 1$ Explain your answer by either proving that f is continuous at $x = 1$ or by explaining why it is not.

4. Given that:

$$\lim_{x \rightarrow 3^+} f(x) = 4, \quad \lim_{x \rightarrow 3^-} f(x) = 2, \quad \lim_{x \rightarrow -2} f(x) = 2, \quad f(3) = 3, \quad f(-2) = 1$$

sketch a possible graph of



5. If $G(t) = \frac{4t}{t+1}$

a) find $f'(x)$ using the definition of derivative.

b) find the equation of the tangent line when $x = 3$.

6. If $f(x) = x^3 - 3x + 5$ find the derivative of the function using the definition of derivative.

7. Find $\frac{dy}{dx}$ for $y = \cot^2(\sin x)$

8. Find $\frac{dy}{dx}$ for $\sin(x+y) = y^2 \cos x$ use implicit differentiation.

9. Find $\frac{dy}{dx}$ for $e^{4y} - \ln y = 2x$ use implicit differentiation.

10. Find $\frac{dy}{dx}$ for $y = 4 \sec^{-1}(x^3 + 1)$

11. Use differentials to approximate Δy when x changes as indicated.

$f(x) = \sqrt{x}$; from $x = 1$ and $x = 1.3$

12. Use logarithmic differentiation to find the derivative $f(x) = x^{\sin x}$

13. A mass on a spring vibrates horizontally on a smooth level surface. Its equation of motion is $x(t) = 8 \sin t$ where t is in seconds and x is in centimeters.

a). Find the velocity and acceleration equations for this spring.

b) Find the position, velocity, and acceleration of the mass at time $t = \frac{2\pi}{3}$. In what direction is it moving at that time? Is it speeding up or slowing down?

14. Gravel is being dumped from a conveyor belt at a rate of $30 \text{ ft}^3 / \text{min}$ and its coarseness is such that it forms a pile in the shape of a cone whose base diameter and height are always equal. How fast is the height of the pile increasing when the pile is 10 ft high?

15. A company needs to run an oil pipeline from an oil rig 25 miles out to sea to a storage tank that is 5 miles inland. The shoreline runs east-west and the tank is 8 miles east of the rig. Assume it costs \$50 thousand per mile to construct the pipeline under water and \$20 thousand per mile to construct the pipeline on land. The pipeline will be built in a straight line to the storage tank. What point on the shoreline should be selected to minimize the total cost of the pipeline?

16. Suppose a finger joint manufacturer sells lug bases for \$250 per unit. If the daily production cost in dollars for x units is $C(x) = 0.1x^2 + 40x + 3600$. How many units of lug bases must the manufacturer sell each day to maximize profit.

17. Given the function $f(x) = x^3 + 3x^2 - 9x$ find each of the following limits, if it exist.

- The critical numbers,
- The local maximums and minimums and indicate which is which,
- The interval(s) where f is increasing,
- The interval(s) where f is decreasing
- The inflection points,
- The interval(s) where f is concave up,
- The interval(s) where f is concave down.

18. Use L'Hospital's rule to find $\lim_{x \rightarrow 0} \frac{\sin x - x}{x^3}$

19. Use Newton's method to find the root of $\cos x - x = 0$, your answer should be accurate to 6 decimal places and please show the value of the first approximation and each successive iteration.